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Calculation programs for machine design

KISSsoft, shaft analysis:

Introduction to DIN 743, October 2000

Shafts and axles, calculation of load capacity

Introduction to DIN 743

Introduction

Purpose

The DIN 743 for strength analysis of shafts and axles is a most helpful analysis method available in the KISSsoft software, [1], for analysis of machine elements. The standard however is available in German only and the theory behind the software KISSsoft is not readily available for non german speaking customers. Therefore, a short introduction to the said standard is given herewith.

The DIN 743

The German standard DIN 743 [2] has been prepared by the German institute for standardisation and the Institut für Maschinenelemente und Maschinenkonstruktion of the technical university of Dresden, Germany. The objective was to make available for the engineering community a standard focusing on strength analysis of shafts and axles. The standard is based on the standard TGL 19340 of the former German Democratic Republic, the VDI 2226 of the Federal Republic of Germany and the FKM guideline compiled by the IMA Dresden, Germany, see [3], [4], [13]. The proof of strength is based on the calculation of a safety factor against fatigue and against static failure. The safety factors have to be higher than a required minimal safety factor. If this condition is fulfilled, proof is delivered.
The standard consists of four parts:

- Part 1: Introduction, analysis method
- Part 2: Stress concentration factors and fatigue notch factors
- Part 3: Materials data
- Part 4: Examples

Limitations

The analytical proof considers bending, tensile/compressive and shear stresses due to torsion. However, shear stresses due to shear forces are not considered, hence use of this standard for short shafts requires caution.
Only the fatigue limit is used in the proof, no proof for finite life strength is delivered. However, an extension is planned, see section 0.
Materials data are based on 10^7 stress cycles with a probability of survival of 97.5%.
The safety factor required in the standard covers only the uncertainty in the analysis procedure. Additional safety factors or an increased safety factor due to uncertainties in the load assumptions and due to the effects of a failure are not defined. They have to be defined by the engineer.

The notch factors for feather keys are questionable since no difference is made for the different key forms.
All loads (bending, tensile/compression, shear) are in phase.
The standard does not cover the calculation of the load acting.
The standard is limited to non-welded steels in the range of –40C° to 150C°. The environment has to be non-corrosive for application of this standard.

Loads

The loads acting on the part are defined by describing the effective load amplitudes and the mean loads (for the fatigue proof) and the maximum acting load (for static proof). These loads are to be calculated according to the nominal stress concept, using standard engineering formulas.

Proof against fatigue failure

Safety

In order to deliver the proof against fatigue failure, the safety S calculated has to be higher than the minimal required safety S_{min}. According to the standard, S_{min} has to be at least 1.2. Uncertainties in the load assumptions and severe effects in case of failure require higher safety factors, to be defined by the engineer. The safety S is calculated from partial safeties according to the following formula:

$$S = \frac{1}{\sqrt{\left(\frac{\sigma_{zda}}{\sigma_{zdADK}} + \frac{\sigma_{ba}}{\sigma_{bADK}}\right)^2 + \left(\frac{\tau_{ta}}{\tau_{tADK}}\right)^2}}$$

where

$\sigma_{zda}, \sigma_{ba}, \tau_{ta}$	Stress amplitudes due to tension/compression, bending, torsion
$\sigma_{zdADK}, \sigma_{bADK}, \tau_{tADK}$	Permissible stress amplitudes, strengths

The form of the above formula is based on the idea of partial safeties for the specific load types (normal stresses / shear stresses) combined in elliptic form.

The stress amplitudes are calculated based on the nominal stress concept, considering the cross section of the shaft (A, I, W_b, W_p) and external loads (moments, forces).

For calculation of the permissible stress amplitudes, see section 0.

Condition for delivery of proof is that

$$S \geq S_{\min} \geq 1.20$$

Two different cases are distinguished:

Case 1: The safety factor is based only on the comparison between actual and permissible stress amplitude, leaving the mean stress on a constant level

Case 2: The safety is based on the assumption that the stress ratio used for calculation of permissible stress amplitude is equal to the stress ratio as it is for the actual stress amplitude

The latter is the more conservative approach and recommended.

Strength of the part

The strength of the part (index ADK) is being calculated from the strength of the un-notched material specimen. It is a nominal stress considering

- Technological size factor $K_1(d_{eff})$: effect of heat treatment (size of shaft), depending on diameter at time of treatment), see section 0
- Geometrical size factor $K_2(d)$: effect of stress gradient on bending strength compared to tensile strength of specimen, see section 0
- Notches, notch factor $\beta_\sigma(d)$, $\beta_\tau(d)$: effect of local stress increasers/notches
- Surface roughness factor $K_{F\sigma}$, $K_{F\tau}$, see section 0
- Surface hardening factor K_V: effect of compressive residual stresses, see section 0

Mean stress sensitivity $\psi_{\sigma K}$ and $\psi_{\tau K}$: Effect of mean stress level on permissible stress amplitude.

Basic assumption is that the ultimate strength R_m or σ_B is based on a reference diameter d_B for which the ultimate strength R_m is tabulated (see part 3 of the standard).

R_m / σ_B may also be estimated from measured Brinell hardness values according to:

$$\sigma_B \approx 0.3 * H_{HB}$$

For a part with diameter $d > d_B$ lower strength applies, the difference being considered using the technological size factor $K_1(d)$. This factor depends on the type of material used and its hardenability / heat treatability:

$$\sigma_B(d) = K_1(d) * \sigma_B(d_B)$$

Where

$\sigma_{zB}(d)$	Strength of the part with diameter d
$\sigma_{zB}(d_B)$	Strength of the specimen with diameter d_B
$K_1(d)$	Technological size coefficient

Based on this ultimate strength of the part, the fatigue strengths are estimated as follows (for bending, tension/compression and shear):

$$\sigma_{bw}(d) = 0.5 * \sigma_B(d)$$
$$\sigma_{zdw}(d) = 0.4 * \sigma_B(d)$$
$$\tau_{tw}(d) = 0.3 * \sigma_B(d)$$

The fatigue strength of the notched part then is (influence of mean stress not yet considered, see section 0), for tension/compression (index zd), bending (index b) and torsion (index t):

$$\sigma_{zdWK} = \frac{\sigma_{zdW}(d_B) * K_1(d_{eff})}{K_\sigma}$$
$$\sigma_{bWK} = \frac{\sigma_{bW}(d_B) * K_1(d_{eff})}{K_\sigma}$$
$$\tau_{tWK} = \frac{\tau_{tW}(d_B) * K_1(d_{eff})}{K_\tau}$$

Where

$K_1(d_{eff})$	Technological size factor
$\sigma_{zdW}, \sigma_{bW}, \tau_{tW}$	Strength of the un-notched specimen with diameter d_B These values are listed in Part 3 of the standard

$$K_\sigma = \left(\frac{\beta_\sigma}{K_2(d)} + \frac{1}{K_{F\sigma}} - 1 \right) * \frac{1}{K_V}$$
$$K_\tau = \left(\frac{\beta_\tau}{K_2(d)} + \frac{1}{K_{F\tau}} - 1 \right) * \frac{1}{K_V}$$

Where

$K_2(d)$	Geometrical size coefficient, see section 0
$\beta_{\sigma,\tau}$	Notch factor for compression/tension, bending and torsion Listed in Part 2 of the standard

Hence, the effect of the notch is considered in the permissible stress and not in the actual stress (calculated as nominal stress).

Influence of mean stress

Two different cases are to be distinguished, see case 1 and case 2 in section 0. In both cases, the strength of the notched part considering the mean stress is a function of the strength of the notched part not considering the mean stress, the mean stress and the mean stress sensitivity:

$$\sigma_{zdADK}, \sigma_{bADK}, \tau_{tADK} = f(\sigma_{zdWK}, \sigma_{bWK}, \tau_{tWK}, \sigma_{mv}, \tau_{mv}, \psi_{zd\sigma K}, \psi_{b\sigma K}, \psi_{\tau K})$$

For case 1, we find:

$$\sigma_{zdADK} = \sigma_{zdWK} - \psi_{zd\sigma K} * \sigma_{mv}$$
$$\sigma_{bADK} = \sigma_{bWK} - \psi_{b\sigma K} * \sigma_{mv}$$
$$\sigma_{zdADK} = \tau_{tWK} - \psi_{\tau K} * \tau_{mv}$$

And for case 2 (limiting conditions not shown, see standard for more details)

$$\sigma_{zdADK} = \frac{\sigma_{zdWK}}{1+\psi_{zd\sigma K} * \frac{\sigma_{mv}}{\sigma_{zda}}}$$

$$\sigma_{bADK} = \frac{\sigma_{bWK}}{1+\psi_{b\sigma K} * \frac{\sigma_{mv}}{\sigma_{ba}}}$$

$$\tau_{tADK} = \frac{\tau_{tWK}}{a+\psi_{\tau K} * \frac{\tau_{mv}}{\tau_{ta}}}$$

Where

$$\sigma_{mv} = \sqrt{(\sigma_{zdm} + \sigma_{bm})^2 + 3 * \tau_{tm}^2}$$

$$\tau_{mv} = \frac{\sigma_{mv}}{\sqrt{3}}$$

The mean stress sensitivity factors ψ themselves depend on the ultimate strength of the material and the alternating strength. According to the formulas used in the standard, the mean stress sensitivity factor is independent of the level of the mean stress, although new findings show that it is higher for low mean stresses and lower for higher mean stresses, see [11].

Static proof

Safety

For this proof, the maximum stress occurring during the lifetime of the part is to be compared to it's strength. The resulting safety is calculated as follows:

$$S = \frac{1}{\sqrt{\left(\frac{\sigma_{zd\max}}{\sigma_{zdFK}} + \frac{\sigma_{b\max}}{\sigma_{bFK}}\right)^2 + \left(\frac{\tau_{t\max}}{\tau_{tFK}}\right)^2}}$$

Note that for the static proof, the effect of notches is not considered.

Strength of the part

In the above formula, strengths of the part (in the denominators) are compared to the acting stresses (in the numerators). The comparisons are than added up in the sense of a elliptic safety criterion. Generally, the yield strength of the part of diameter d is not known (e.g. from measurement) and is to be estimated from the values of the specimen with diameter d_B. The yield strengths of the part, generally written as $\sigma_s(d)$ (yield strength for a part of diameter d) are calculated as follws:

$$\sigma_{zdFK} = K_1(d_{eff}) * K_{2F} * \gamma_F * \sigma_s(d_B)$$

$$\sigma_{bFK} = K_1(d_{eff}) * K_{2F} * \gamma_F * \sigma_s(d_B)$$

$$\tau_{tFK} = K_1(d_{eff}) * K_{2F} * \gamma_F * \sigma_s(d_B) / \sqrt{3}$$

Where

- $K_1(d_{eff})$ is the technological size coefficient
- K_{2F} is the static support coefficient depending on the presence of a hardened outer layer of the material
- γ_F is a coefficient for increasing the yield strength due to a multi-axial stress state in a notch
- $\sigma_S(d_B)$ is the yield strength of the specimen with diameter d_B

Remarks

Notch effects

The notch factor β is defined through the permissible stress amplitude against fatigue failure of the un-notched specimen of diameter d compared to the one of the notched part:

$$\beta_\sigma = \frac{\sigma_{zd,bW}(d)}{\sigma_{zd,bWK}}$$

$$\beta_\tau = \frac{\tau_{tW}(d)}{\tau_{tWK}}$$

Whereas the form factor α is the coefficient of the stress value in the notch compared to the stress value in the un-disturbed cross section (nominal stress). Hence, whereas the form factor is a function only of the geometry of the part, the notch factor β is also a function of the material and the stress state. The notch factor β can be calculated from the form factor α as follows:

$$\beta_{\sigma,\tau} = \frac{\alpha_{\sigma,\tau}}{n}$$

Where n is the support coefficient to be calculated from the materials strength and the stress gradient in the notch. The stress gradient can either be calculated by means of FEM or estimated through formulas given in the standard.

Different notch factors are given for all three load types considered (bending, tension/compression and torsion),

Influence of size

The influence of the size of a part on it's strength is to be considered in three factors:

1) Technological size coefficient $K_1(d_{eff})$: With this factor the fact that the effect of hardening / heat treatment and hence the strength is reduced with increasing part diameter. This coefficient is independent of the type of load (tension/compression, bending, shear). The coefficient is to be estimated using the effective diameter during heat treatment. The coefficient is to be considered if the strength of the part is not measured but calculated from the strength of the specimen as tabulated in the standard.
2) Geometrical size coefficient $K_2(d)$: With this factor, the effect that the strength against bending converges towards the strength against tension/compression with increasing part diameter and

that the strength against shear due to torsion is also being reduced. This is due to the decreasing stress gradient with increasing part diameter. With the decreasing stress gradient, the support coefficient n also decreases.

3) Geometrical size coefficient $K_3(d)$: Same effect as in 2) but here for the notched part.

Influence of surface hardening

With the coefficient K_V effects due to surface hardening depending on the procedure applied are considered. The underlying effects covered are the surface hardness and induced compressive stresses. It is recommended to use the lower range of the tabulated values or to conduct experiments if the higher values are to be used, see for example [6].

Finite life calculation

The standard DIN 743 covers only the proof against fatigue failure for infinite life. However, an extension to finite life calculation is planned, see [12], [8]. With this extension, load spectrums can be considered. Based on the load spectrum, a damage equivalent load is calculated. Based on this damage equivalent load, using a S-N curve, the lifetime is estimated or the proof for infinite life is delivered.

The damage equivalent load amplitude is calculated as follows:

$$\sigma_a = \sigma_{a1} / f_{koll}$$

Where

σ_a is the damage equivalent load amplitude
σ_{a1} is the highest load amplitude of the collective
f_{koll} describes the shape of the load sepctrum, calculated according to different variations of Miners rule, see [12]

The permissible stress amplitude for a required life time is calculated as follows:

$$\sigma_{ANK} = \sqrt[q]{\frac{N_D}{N_L}} * \sigma_{ADK}$$

Where

σ_{ANK} is the permissible stress amplitude for finite life
σ_{ADK} is the permissible stress amplitude for infinite life
$N_D = 1*10^6$ cycles
N_L is the required life (cycles), $1*10^3<N_L<N_D$
q=5 for bending and tension/compression, q=8 for torsion

Comparison to other analysis methods

In [7], [9] and [5], comparisons of the analysis method according to DIN 743 with other methods, FKM-Richtlinie [13], Nieman [14] and Hänchen + Decker [15], are described.

In comparison with the FKM-guideline, the following differences are found:

- In the FKM guideline, shear stress due to shear forces are considered
- The minimal safety factors required also depend on the material, load assumptions and effect of failure

- High temperature is considered in static proof
- Differences in notch factors

A comparison with the other two methods cited shows that the complete analysis procedure is quite different.
All authors agree that the deviations in the results strongly depend on the notch considered. No clear tendencies were found when comparing the results, but the usefulness of the standard and the soundness of the design resulting from its use were confirmed in all cases.

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